Multiplication and Division (including Fractions)

The national curriculum for mathematics aims to ensure that all pupils:

1. become **fluent** in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.
2. **reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
3. can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence. By the end of Year 6, children should be able to choose the most appropriate approach to solve a problem: making a choice between using jottings (an extended written method), an efficient written method or a mental method.

The policy outlines concrete, pictorial and abstract practices. When children are secure and confident using a concrete or pictorial method they should be moved on accordingly. An example of a resource has been given but other representations, concrete or pictorial, should be used when appropriate. This will assist deeper understanding.

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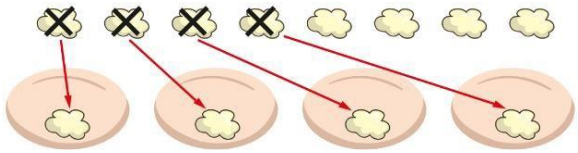
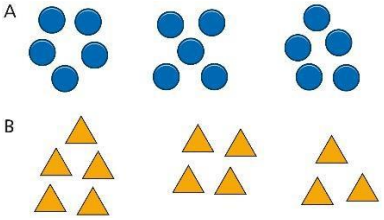
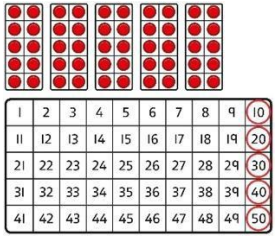
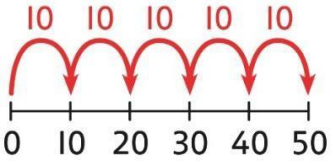
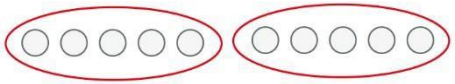
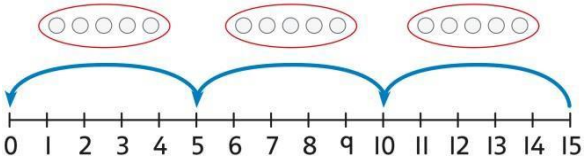
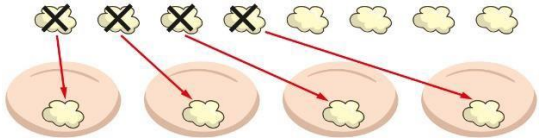
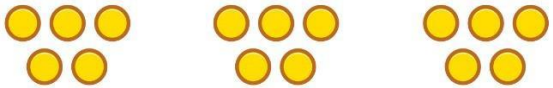
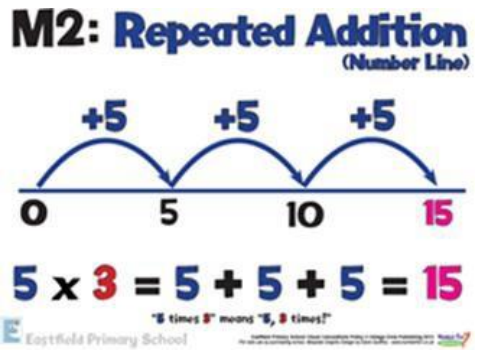
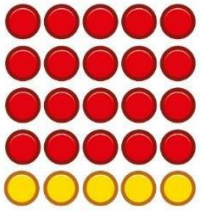
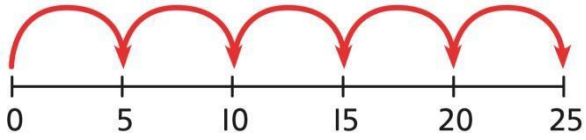
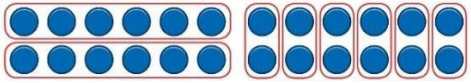
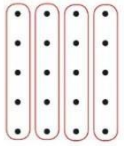
KEY STAGE 1
Multiplication and Division Children develop an awareness of equal groups and link this with counting in equal steps, starting with 2s, 5s and 10s. In Year 2, they learn to connect the language of equal groups with the mathematical symbols for multiplication and division. They learn how multiplication and division can be related to repeated addition and repeated subtraction to find the answer to the calculation. In this key stage, it is vital that children explore and experience a variety of strong images and manipulative representations of equal groups, including concrete experiences as well as abstract calculations. Children begin to recall some key multiplication facts, including doubles, and an understanding of the 2, 5 and 10 times-tables and how they are related to counting. **Fractions** In Year 1, children encounter halves and quarters, and link this with their understanding of sharing. They experience key spatial representations of these fractions, and learn to recognise examples and non-examples, based on their awareness of equal parts of a whole. In Year 2, they develop an awareness of unit fractions and experience non-unit fractions, and they learn to write them and read them in the common format of numerator and denominator.
Key language: group, share, equal, equals, is equal to, groups, equal groups, times, multiply, multiplied by, divide, share, shared equally, times-table, **dividend, divisor, quotient, product, factors**

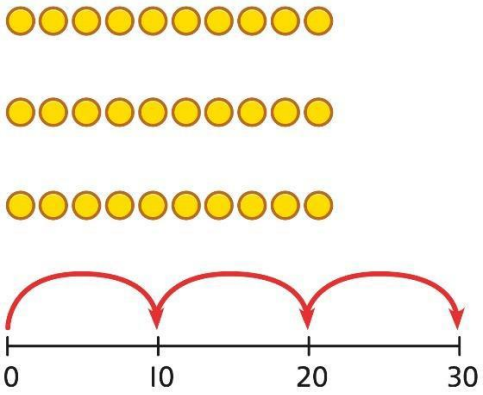
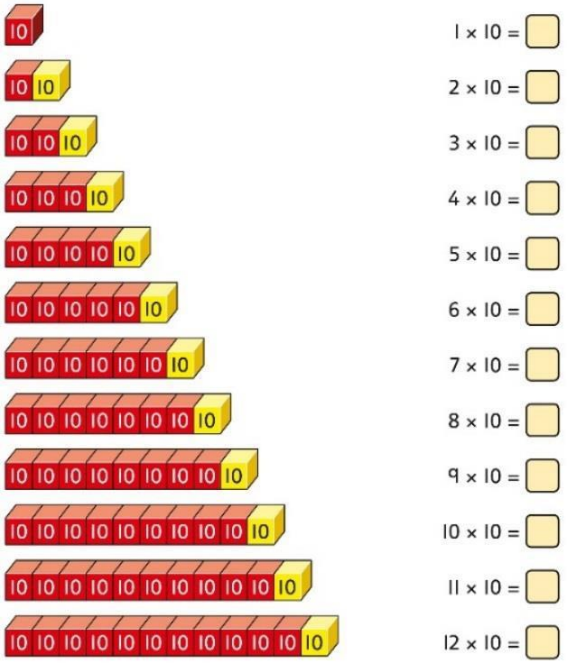
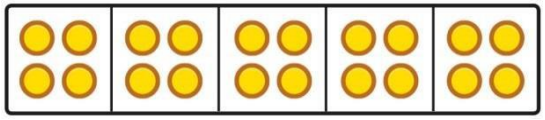
	Concrete	Pictorial	Abstract
	Foundation Multiplication		
Solve problems involving doubling up to 10.	**Recognising and make two equal groups** Children arrange objects into two equal groups. A range of objects / representations should be used.   	Children draw two equal groups. 	
	Foundation Division		
Solve problems involving halving up to 10.	**Grouping**  *There are 10 children altogether.* *There are 2 in each group.* *There are 5 groups.*		

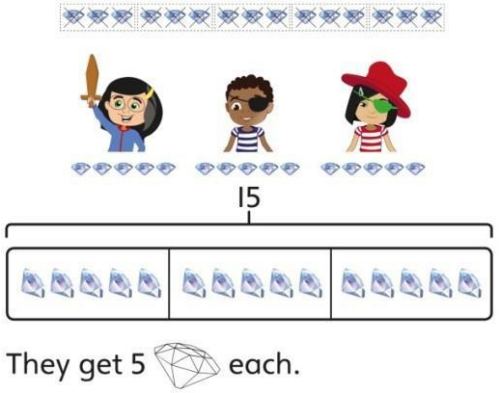
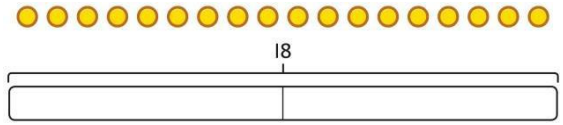
Solve problems involving sharing up to 10	**Sharing** Share a set of objects into equal parts and work out how many are in each part**.** 		

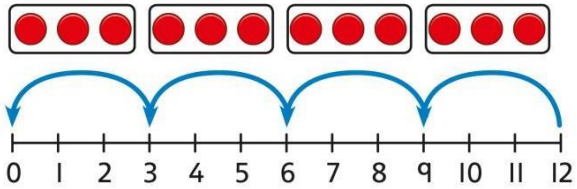
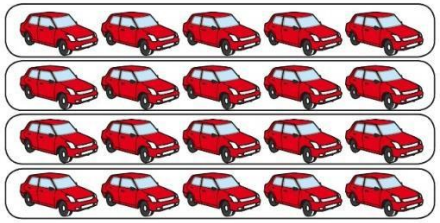
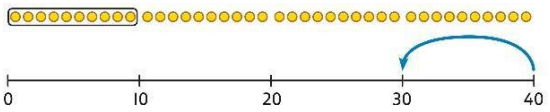
	Concrete	Pictorial	Abstract
	Year 1 Multiplication		
	Recognising and making equal groups Children arrange objects in equal and unequal groups and understand how to recognise whether they are equal. 	**Recognising and making equal groups** Children draw and represent equal and unequal groups. 	**Describe equal groups using words** *Three equal groups of 4.* *Four equal groups of 3.*
	Finding the total of equal groups by counting in 2s, 5s and 10s  There are 5 pens in each pack … 5…10…15…20…25…30…35…40…	**Finding the total of equal groups by counting in 2s, 5s and 10s** 100 squares and ten frames support counting in 2s, 5s and 10s. 	**Finding the total of equal groups by counting in 2s, 5s and 10s** Use a number line to support repeated addition through counting in 2s, 5s and 10s. 
	Year 1 Division		
	Grouping Learn to make equal groups from a whole and find how many equal groups of a certain size can be made.	**Grouping** Represent a whole and work out how many equal groups.	**Grouping** Children may relate this to counting back in steps of 2, 5 or 10.

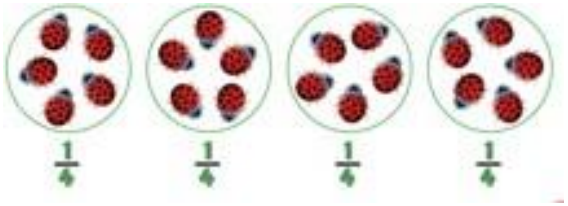
	Sort a whole set people and objects into equal groups.  *There are 10 children altogether.* *There are 2 in each group.* *There are 5 groups.*	 *There are 10 in total.* *There are 5 in each group.* *There are 2 groups.*	
	Sharing Share a set of objects into equal parts and work out how many are in each part**.** 	**Sharing** Sketch or draw to represent sharing into equal parts. This may be related to fractions. 	**Sharing** *10 shared into 2 equal groups gives 5 in each group.*

	Concrete	Pictorial	Abstract
	Year 2 Multiplication		
Equal groups and repeated addition	Recognise equal groups and write as repeated addition and as multiplication.  *3 groups of 5 chairs* *15 chairs altogether*	Recognise equal groups using standard objects such as counters and write as repeated addition and multiplication.  *3 groups of 5* *15 in total*	
Using arrays to represent multiplication	Understand the relationship between arrays, multiplication and repeated addition.  *4 groups of 5*	 *4 groups of 5 … 5 groups of 5*	 *5 × 5 = 25*
Commutativity	 *I can see 6 groups of 3.* *I can see 3 groups of 6.*	Use counters to visualise commutativity.  *This is 2 groups of 6 and 6 groups of 2.*	 *and 5 × 4 = 20*  *4 + 4 + 4 + 4 + 4 = 20* *5 + 5 + 5 + 5 = 20* *4 × 5 = 20*

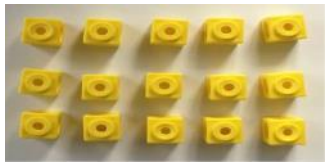
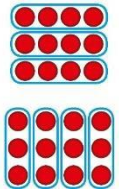
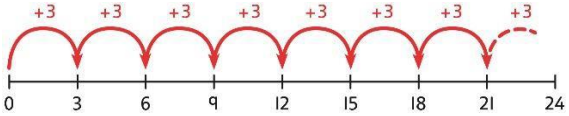
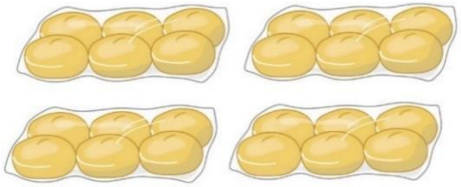
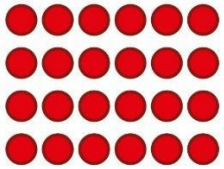
Learning ×2, ×5 and ×10 table facts	 *3 groups of 10 … 10, 20, 30* *3 × 10 = 30*	 *10 + 10 + 10 = 30* *3 × 10 = 30*	 *5 × 10 = 50* *6 × 10 = 60*
Year 2 Division			
Sharing equally			Use a bar model to support understanding of the division.

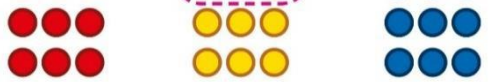
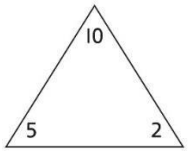
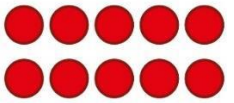
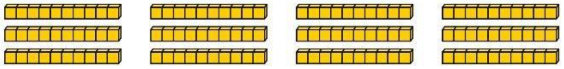
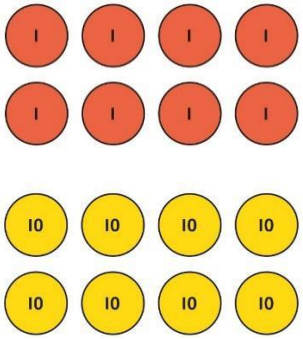
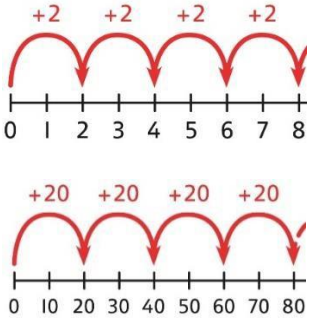
	12 shared equally between 2. *They get 6 each.* Start to understand how this also relates to grouping. To share equally between 3 people, take a group of 3 and give 1 to each person. Keep going until all the objects have been shared  *15 shared equally between 3.* *They get 5 each.*	*20 shared into 5 equal parts.* *There are 4 in each part.*	 *18 ÷ 2 = 9*
Grouping equally	 *8 divided into 4 equal groups.* *There are 2 in each group.*	Understand the relationship between grouping and the division statements.	Understand how to relate division by grouping to repeated subtraction.

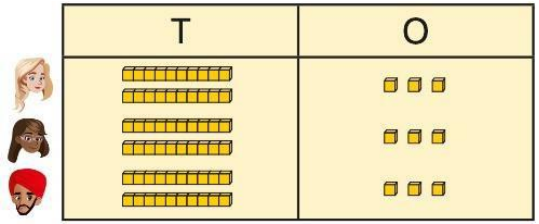
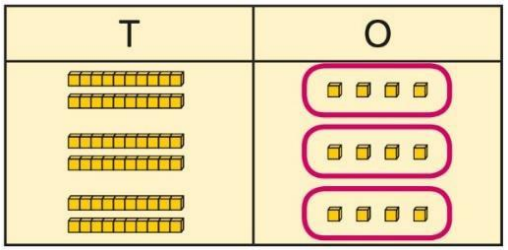
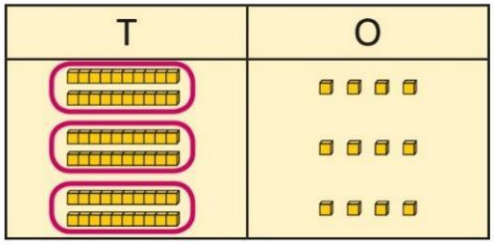
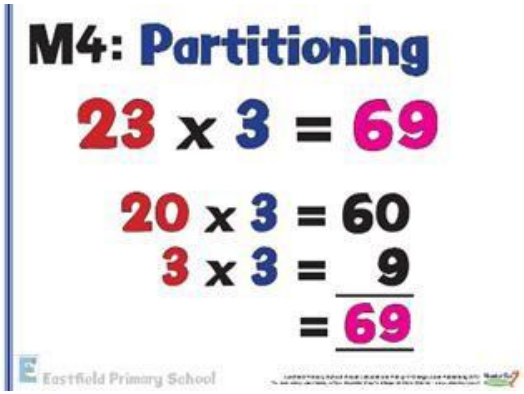
		12 ÷ 3 = 4  12 ÷ 4 = 3  12 ÷ 6 = 2  12 ÷ 2 = 6 	 There are 4 groups now. *12 divided into groups of 3.* *12 ÷ 3 = 4* *There are 4 groups.*
Using known times-tables to solve divisions	Understand the relationship between multiplication facts and division.  *4 groups of 5 cars is 20 cars in total.* *20 divided by 4 is 5.*	Link equal grouping with repeated subtraction and known times-table facts to support division.  *40 divided by 4 is 10.*	1 × 10 = 10 2 × 10 = 20 3 × 10 = 30 4 × 10 = 40 5 × 10 = 50 6 × 10 = 60 7 × 10 = 70 8 × 10 = 80  I used the 10 times-table to help me. 3 × 10 = 30. *I know that 3 groups of 10 makes 30, so I know that 30 divided by 10 is 3.* *3 × 10 = 30 so 30 ÷ 10 = 3*

Understand how fractions and division link. *(Find a 1/3, ¼ or 1/2 of a quantity).*	Share the cakes into 4 equal groups.  ¼ of 8 = 4	Children will draw the circles and share the whole number between them.  Draw the groups and share the amount equally.	Children will be able to visually see the link between division and fractions and will be able to calculate the answer mentally. **¼ of 20 = 5** **20 divided by 4 = 5** **1/3 of 9 = 3** **9 divided by 3 = 3**

LOWER KEY STAGE 2
Multiplication and Division Children build a solid grounding in times-tables, understanding the multiplication and division facts in tandem. As such, they should be as confident knowing that 35 divided by 7 is 5 as knowing that 5 times 7 is 35. Children develop key skills to support multiplication methods: unitising, commutativity, and how to use partitioning effectively. Unitising allows children to use known facts to multiply and divide multiples of 10 and 100 efficiently. Commutativity gives children flexibility in applying known facts to calculations and problem solving. An understanding of partitioning allows children to extend their skills to multiplying and dividing 2- and 3-digit numbers by a single digit. Children develop column methods to support multiplications in these cases. For successful division, children will need to make choices about how to partition. For example, to divide 423 by 3, it is effective to partition 423 into 300, 120 and 3, as these can be divided by 3 using known facts. Children will also need to understand the concept of remainder, in terms of a given calculation and in terms of the context of the problem. **Fractions** Children develop the key concept of equivalent fractions, and link this with multiplying and dividing the numerators and denominators, as well as exploring the visual concept through fractions of shapes. Children learn how to find a fraction of an amount and develop this with the aid of a bar model and other representations alongside. in Year 3, children develop an understanding of how to add and subtract fractions with the same denominator and find complements to the whole. This is developed alongside an understanding of fractions as numbers, including fractions greater than 1. In Year 4, children begin to work with fractions greater than 1. Decimals are introduced, as tenths in Year 3 and then as hundredths in Year 4. Children develop an understanding of decimals in terms of the relationship with fractions, with dividing by 10 and 100, and with place value.
Key language: partition, place value, tens, hundreds, thousands, column method, whole, part, equal groups, sharing, grouping, **dividend, divisor, quotient, product, factors, denominator, numerator**

	Concrete	Pictorial	Abstract
Year 3 Multiplication			
Understanding equal grouping and repeated addition	 *I can see 3 groups of 5.* *I can see 5 groups of 3.*	 *This is 3 groups of 4.* *This is 4 groups of 3.*	Children understand the link between repeated addition and multiplication.  *8 groups of 3 is 24.* *3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 = 24* *8 × 3 = 24*
Using commutativity to support understanding of the times-tables	  *There are 6 groups of 4 pens.* *There are 4 groups of 6 bread rolls.* *I can use 6 × 4 = 24 to work out both totals.*	 *6 × 4 = 24* 4 × 6 = 24	**I need to work out 4 groups of 7.** **I know that 7 × 4 = 28** **so, I know that** **4 groups of 7 = 28** **and** **7 groups of 4 = 28.**

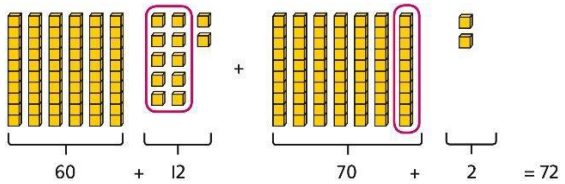
Understanding and using ×3, ×2, ×4 and ×8 tables.	Children learn the times-tables as 'groups of' but apply their knowledge of commutativity.   *I can use the ×3 table to work out how many keys.* *I can also use the ×3 table to work out how many batteries.*	Children understand how the ×2, ×4 and ×8 tables are related through repeated doubling.  3 × 2 = 6 3 × 4 = 12 3 × 8 = 24	Children understand the relationship between related multiplication and division facts in known times-tables.   *2 × 5 = 10* *5 × 2 = 10* *10 ÷ 5 = 2* *10 ÷ 2 = 5*
Using known facts to multiply 10s, for example 3 × 40	*Make 4 groups of 3 ones.*  *Make 4 groups of 3 tens.*  *What is the same?* *What is different?*	 *4 groups of 2 ones is 8 ones.* *4 groups of 2 tens is 8 tens.* *4 × 2 = 8* *4 × 20 = 80*	 *4 × 2 = 8* 4 × 20 = 80

Multiplying a 2-digit number by a 1-digit number	*Each person has 23 flowers.* *Each person has 2 tens and 3 ones.*  *There are 3 groups of 2 tens.* *There are 3 groups of 3 ones.* Use place value equipment to model the multiplication context.  *There are 3 groups of 3 ones.* *There are 3 groups of 2 tens.*	Use place value to support how partitioning is linked with multiplying by a 2-digit number. *3 × 24 = ?*  *3 × 4 = 12*  *3 × 20 = 60* *60 + 12 = 72* 3 × 24 = 72	***4 × 13 = ?*** ***4 × 3 = 12*** ***4 × 10 = 40*** ***12 + 40 = 52*** ***4 × 13 = 52*** 

Multiplying a 2-digit number by a 1-digit number, expanded column method

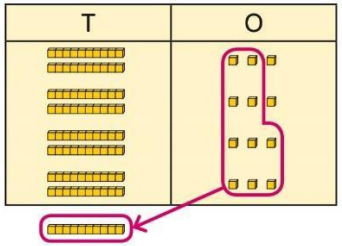
3 × 24 = ?

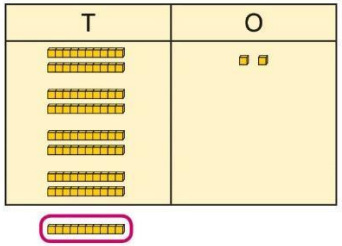
3 × 20 = 60
3 × 4 = 12



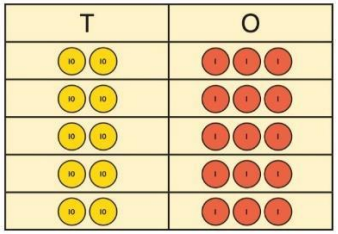
3 × 24 = 60 + 12
3 × 24 = 70 + 2
3 × 24 = 72

4 × 23 = ?

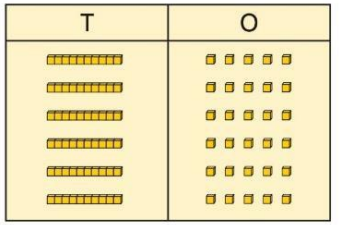




4 × 23 = 92



5 × 23 = ?
5 × 3 = 15
5 × 20 = 100
5 × 23 = 115



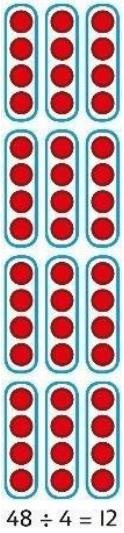
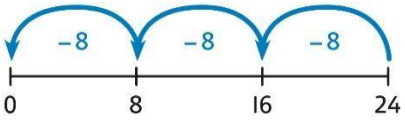
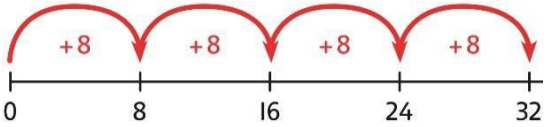
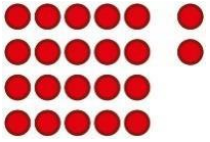
T	O
1	5
×	6
+	

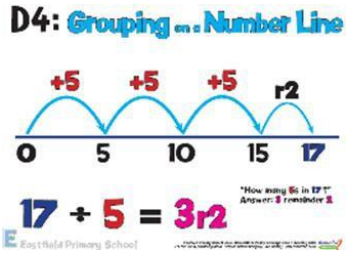
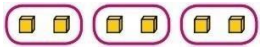
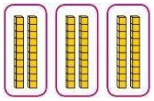
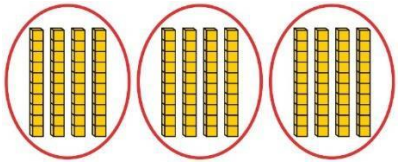
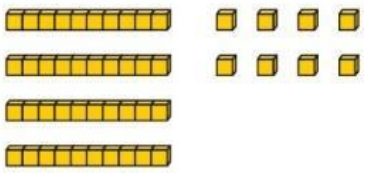
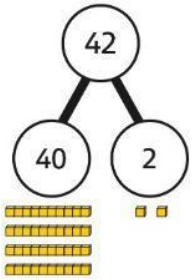
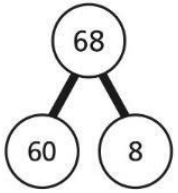
6 × 5
6 × 10

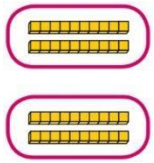
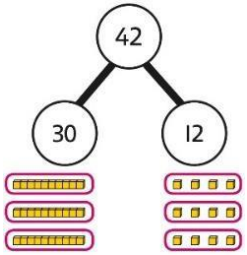
5 × 28 = ?

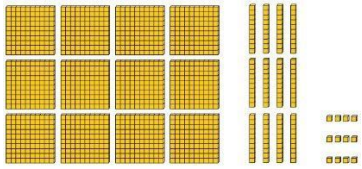
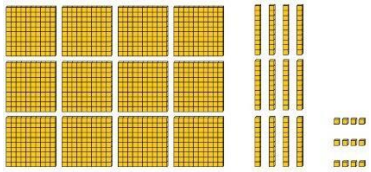
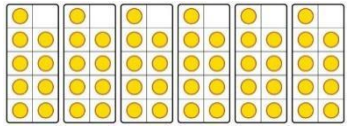
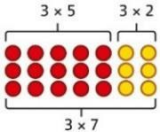
T	O	
	2	8
×		5
	4	0
	1	0 0
	1	4 0

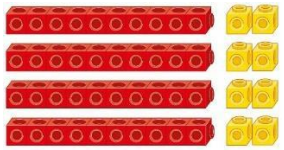
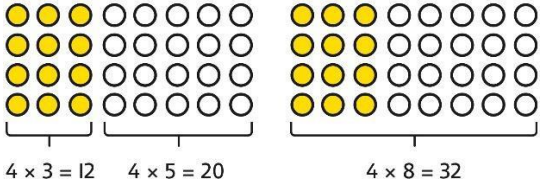
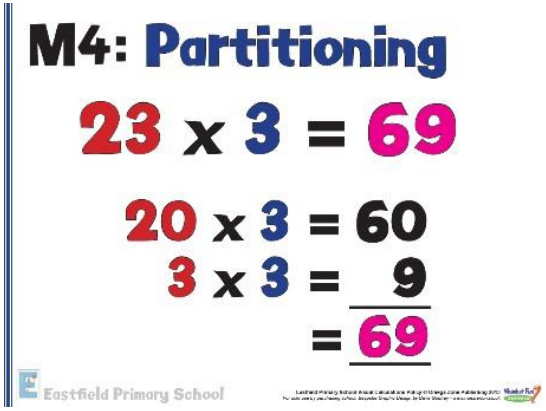
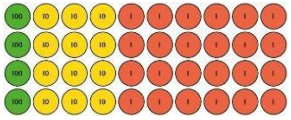
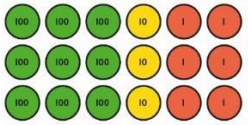
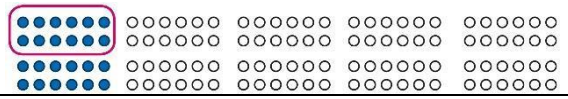
5 × 8
5 × 20

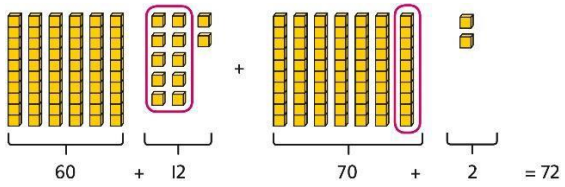
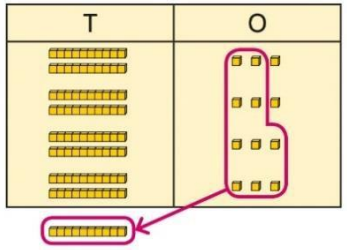
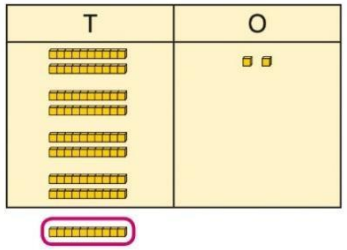
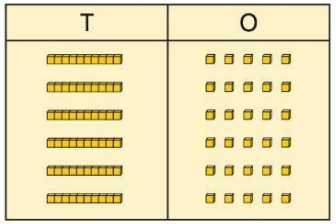
Year 3 Division			
Using times-tables knowledge to divide	 *24 divided into groups of 8.* *There are 3 groups of 8.*	 48 ÷ 4 = 12 *48 divided into groups of 4.* *There are 12 groups.* *4 × 12 = 48* 48 ÷ 4 = 12	*I need to work out 30 shared between 5.* *I know that 6 × 5 = 30* *so I know that 30 ÷ 5 = 6.* *24 ÷ 4 = 6* *24 ÷ 6 = 4* Children understand how division is related to both repeated subtraction and repeated addition.  *24 ÷ 8 = 3*  *32 ÷ 8 = 4*
Understanding remainders	Use equipment.  *There are 13 sticks in total.* *There are 3 groups of 4, with 1 remainder.*	Use images.  *22 ÷ 5 = 4 remainder 2*	*22 ÷ 5 = ?* *3 × 5 = 15* *4 × 5 = 20* *5 × 5 = 25 … this is larger than 22* *So, 22 ÷ 5 = 4 remainder 2*

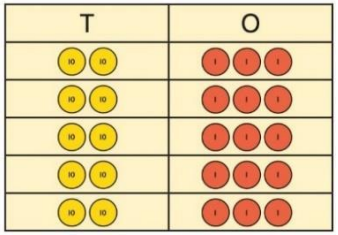
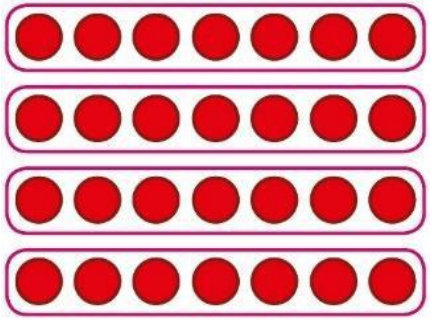
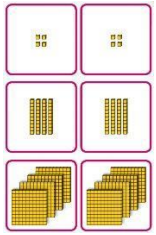
			
Using known facts to divide multiples of 10	*Make 6 ones divided by 3.*  *Now make 6 tens divided by 3.*  What is the same? What is different?	Divide multiples of 10 by unitising.  *12 tens shared into 3 equal groups.* 4 tens in each group.	Using known times-tables. *180 ÷ 3 = ?* *180 is 18 tens.* *18 divided by 3 is 6.* *18 tens divided by 3 is 6 tens.* *18 ÷ 3 = 6* 180 ÷ 3 = 60
2-digit number divided by 1-digit number, no remainders	Children explore dividing 2-digit numbers by using place value equipment.  *48 ÷ 2 = ?*		 *60 ÷ 2 = 30* *8 ÷ 2 = 4* *30 + 4 = 34* *68 ÷ 2 = 34*

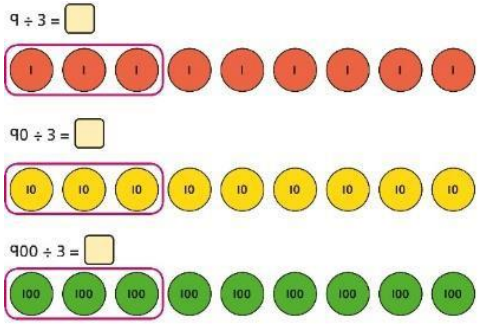
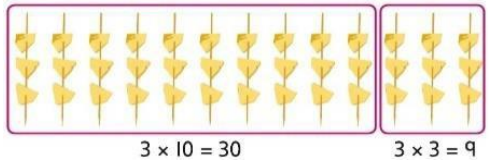
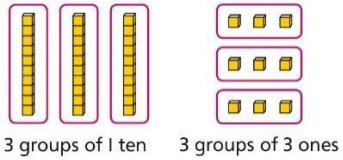
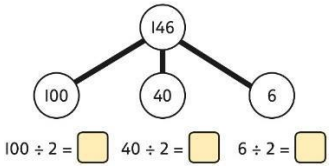
	First divide the 10s.  *Then divide the 1s.* 	*I need to partition 42 differently to divide by 3.*  *42 = 30 + 12* *42 ÷ 3 = 14*	Children partition flexibly to divide where appropriate. *42 ÷ 3 = ?* *42 = 40 + 2* *I need to partition 42 differently to divide by 3.* *42 = 30 + 12* *30 ÷ 3 = 10* *12 ÷ 3 = 4* *10 + 4 = 14* *42 ÷ 3 = 14*
2-digit number divided by 1-digit number, with remainders	*Make 29 from place value equipment. Share it into 2 equal groups*.  *There are two groups of 14 and 1 remainder.*	Use place value equipment to understand the concept of remainder in division. *29 ÷ 2 = ?*  *29 ÷ 2 = 14 remainder 1*	Partition to divide, understanding the remainder in context. *67 children try to make 5 equal lines.* *67 = 50 + 17* *50 ÷ 5 = 10* *17 ÷ 5 = 3 remainder 2* *67 ÷ 5 = 13 remainder 2* *There are 13 children in each line and 2 children left out.*

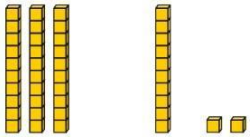
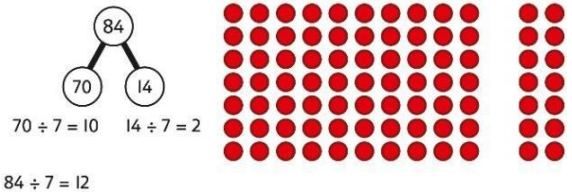
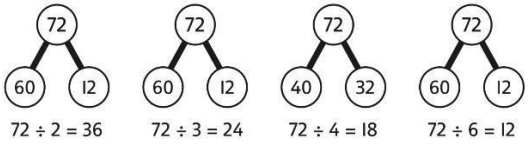
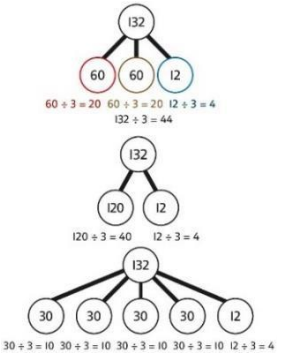
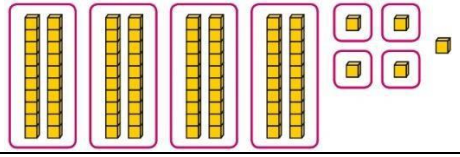
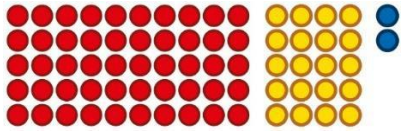
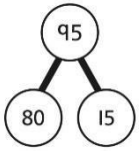
	Concrete	Pictorial	Abstract
	Year 4 Multiplication		
Multiplying by multiples of 10 and 100	 *3 groups of 4 ones is 12 ones.* *3 groups of 4 tens is 12 tens.* *3 groups of 4 hundreds is 12 hundreds.*	 *3 × 4 = 12* *3 × 40 = 120* *3 × 400 = 1,200*	*4 × 7 = 28* *4 × 70 = 280* *40 × 7 = 280* *4 × 700 = 2,800* *400 × 7 = 2,800*
Understanding times-tables up to 12 × 12	Understand the special cases of multiplying by 1 and 0.  5 × 1 = 5 5 × 0 = 0	Represent the relationship between the ×9 table and the ×10 table.  Represent the ×11 table and ×12 tables in relation to the ×10 table.  *2 × 11 = 20 + 2* *3 × 11 = 30 + 3* *4 × 11 = 40 + 4*  4 × 12 = 40 + 8	Understand links between the ×3 table, ×6 table and ×9 table *5 × 6 is double 5 × 3* ×5 table and ×6 table *I know that 7 × 5 = 35* *so I know that 7 × 6 = 35 + 7.* ×5 table and ×7 table *3 × 7 = 3 × 5 + 3 × 2*  ×9 table and ×10 table *6 × 10 = 60* *6 × 9 = 60 − 6*

Distributive law **(Partitioning)**	Make multiplications by partitioning. *4 × 12 is 4 groups of 10 and 4 groups of 2.*  *4 × 12 = 40 + 8*	Understand how multiplication and partitioning are related through addition.  *4 × 3 = 12* *4 × 5 = 20* *12 + 20 = 32* *4 × 8 = 32*	
Column multiplication for 2- and 3-digit numbers multiplied by a single digit	*Make 4 × 136 using equipment.*  *I can work out how many 1s, 10s and 100s.* *There are 4 × 6 ones…* *24 ones* *There are 4 × 3 tens …* *12 tens* *There are 4 × 1 hundreds …* *4 hundreds* *24 + 120 + 400 = 544*	Use place value equipment alongside a column method for multiplication of up to 3-digit numbers by a single digit.  3 1 2 × 3 9 3 6	3 1 2 × 3 9 3 6 Use the formal column method for up to 3-digit numbers multiplied by a single digit. Understand how the long multiplication method links to the short multiplication method. 2 3 × 5 1 5 1 0 0 1 1 5 2 3 × 5 1 1 5 1
Multiplying more than two numbers			*24 × 5 = 12 × 2 × 5*

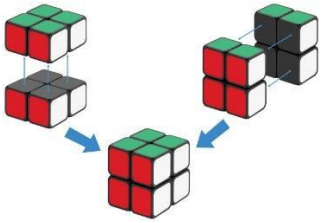
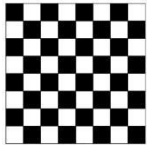
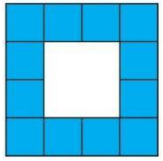
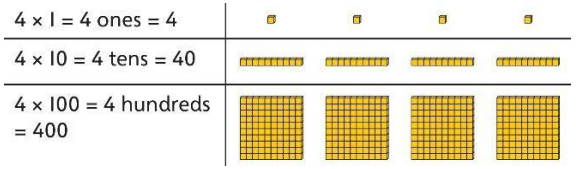
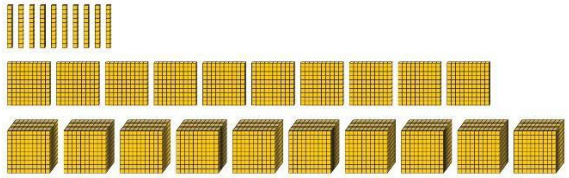
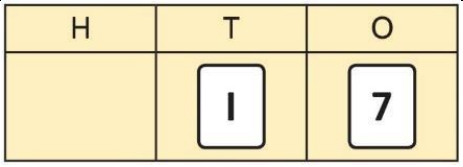
	 *Each sheet has 2 × 5 stickers.* *There are 3 sheets.* *There are 5 × 2 × 3 stickers in total.* $5 \times 2 \times 3 = 30$ $10 \times 3 = 30$	*2 × 6 × 10 = 120* *12 × 10 = 120* *10 × 6 × 2 = 120* 60 × 2 = 120	$12 \times 2 \times 5 =$ $12 \times 10 = 120$ So, $24 \times 5 = 120$
Multiplying a 2-digit number by a 1-digit number, expanded column method	Use place value equipment to model how 10 ones are exchanged for a 10 in some multiplications. *3 × 24 = ?* *3 × 20 = 60* *3 × 4 = 12*  	  Understand that multiplications may require an exchange of 1s for 10s, and 10s for 100s. *4 × 23 = ?* *4 × 23 = 92* *5 × 23 = ?*	 T O 1 5 × 6 + 6 × 5 6 × 10 *5 × 28 = ?* T O 2 8 × 5 4 0 1 0 0 1 4 0 5 × 8 5 × 20

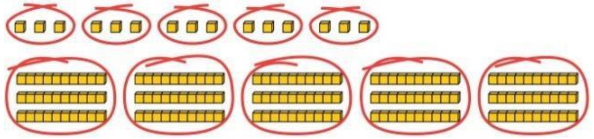
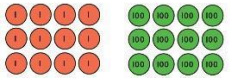
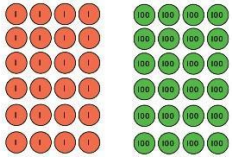
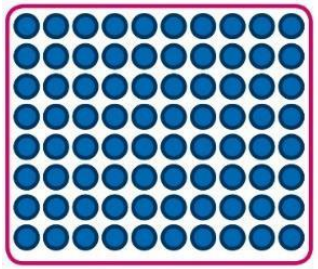
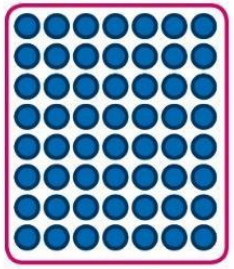
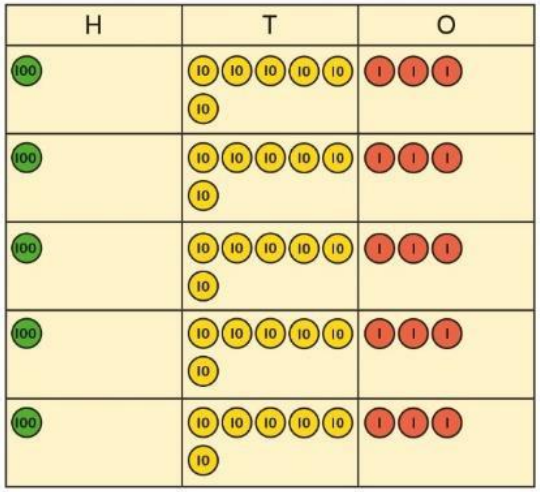
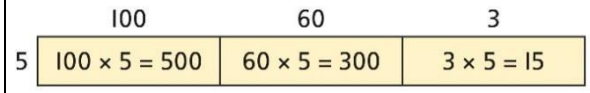
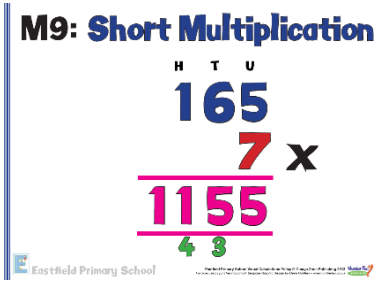
	3 × 24 = 60 + 12 *3 × 24 = 70 + 2* *3 × 24 = 72*	*5 × 3 = 15* *5 × 20 = 100* *5 × 23 = 115* 	
Year 4 Division			
Understanding the relationship between multiplication and division, including times-tables	 *4 × 6 = 24* *24 is 6 groups of 4.* *24 is 4 groups of 6.* *24 divided by 6 is 4.* *24 divided by 4 is 6.*	 28 ÷ 7 = 4	*I know that 5 × 7 = 35 so I know all these facts:* *5 × 7 = 35* *7 × 5 = 35* *35 = 5 × 7* *35 = 7 × 5* *35 ÷ 5 = 7* *35 ÷ 7 = 5* *7 = 35 ÷ 5* 5 = 35 ÷ 7
Dividing multiples of 10 and 100 by a single digit	Use place value equipment to understand how to use unitising to divide. 	Represent divisions using place value equipment.	Use known facts to divide 10s and 100s by a single digit. *15 ÷ 3 = 5* *150 ÷ 3 = 50* *1500 ÷ 3 = 500*

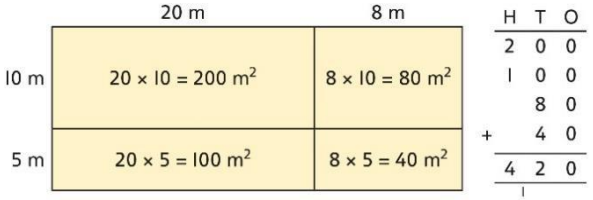
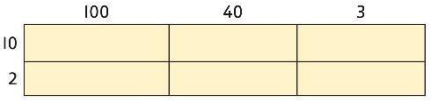
	8 ones divided into 2 equal groups *4 ones in each group* *8 tens divided into 2 equal groups* *4 tens in each group* *8 hundreds divided into 2 equal groups* *4 hundreds in each group*	 *9 ÷ 3 = 3* *9 tens divided by 3 is 3 tens.* *9 hundreds divided by 3 is 3 hundreds.*	
Dividing 2-digit and 3-digit numbers by a single digit by partitioning into 100s, 10s and 1s	*39 ÷ 3 = ?*  *39 = 30 + 9* *30 ÷ 3 = 10* *9 ÷ 3 = 3* *39 ÷ 3 = 13*	*39 ÷ 3 = ?*  *39 = 30 + 9* *30 ÷ 3 = 10* *9 ÷ 3 = 3* *39 ÷ 3 = 13*	*142 ÷ 2 = ?*  *100 ÷ 2 = 50* *40 ÷ 2 = 20* *6 ÷ 2 = 3* *50 + 20 + 3 = 73* 142 ÷ 2 = 73
Dividing 2-digit and 3-	*42 ÷ 3 = ?*	*84 ÷ 7 = ?*	Make decisions about appropriate partitioning based on the division required.

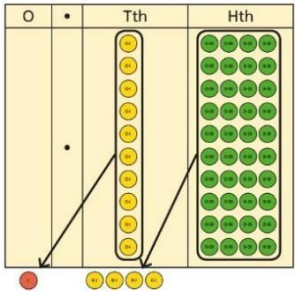
digit numbers by a single digit, using flexible partitioning	*I will split it into 30 and 12, so that I can divide by 3 more easily.* 	*I will partition into 70 and 14 because I am dividing by 7.*  84 ÷ 7 = 12	 Understand that different partitions can be used to complete the same division. 
Understanding remainders	Use place value equipment to find remainders. *85 shared into 4 equal groups* *There are 24, and 1 that cannot be shared.* 	Represent the remainder as the part that cannot be shared equally.  72 ÷ 5 = 14 remainder 2	 *80 ÷ 4 = 20* *12 ÷ 4 = 3* 95 ÷ 4 = 23 remainder 3

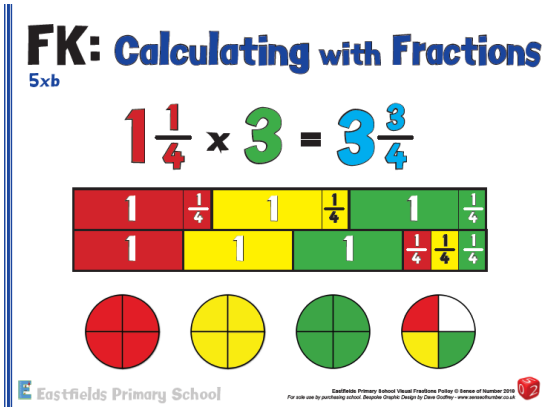
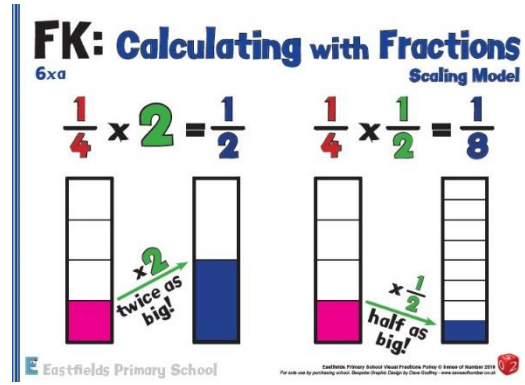
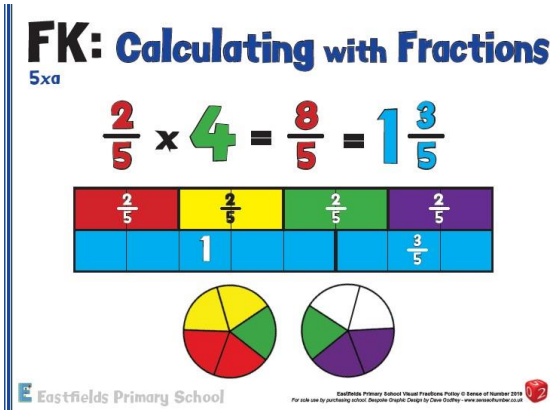
UPPER KEY STAGE 2
Multiplication and Division Building on their understanding, children develop methods to multiply up to 4-digit numbers by single-digit and 2-digit numbers. Children develop column methods with an understanding of place value, and they continue to use the key skill of unitising to multiply and divide by 10, 100 and 1,000. Written division methods are introduced and adapted for division by single-digit and 2-digit numbers and are understood alongside the area mode (grid method) and place value. In Year 6, children develop a secure understanding of how division is related to fractions. Multiplication and division of decimals are also introduced and refined in Year 6. **Fractions** Children find fractions of amounts, multiply a fraction by a whole number and by another fraction, divide a fraction by a whole number, and add and subtract fractions with different denominators. Children become more confident working with improper fractions and mixed numbers and can calculate with them. Understanding of decimals with up to 3 decimal places is built through place value and as fractions, and children calculate with decimals in the context of measure as well as in pure arithmetic. Children develop an understanding of percentages in relation to hundredths, and they understand how to work with common percentages: 50%, 25%, 10% and 1%.
Key language: decimal, column methods, exchange, partition, mental method, ten thousand, hundred thousand, million, factor, multiple, prime number, square number, cube number, **dividend, divisor, quotient, product, factors, denominator, numerator**

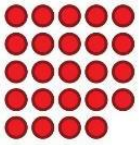
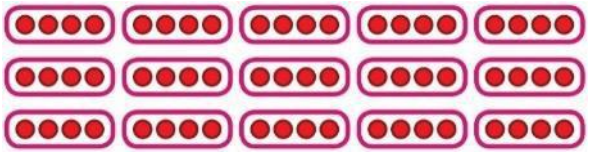
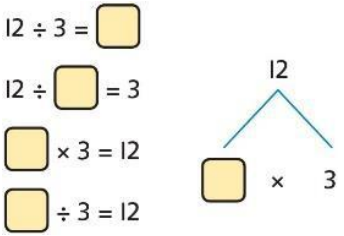
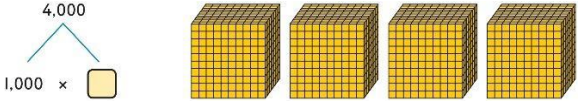
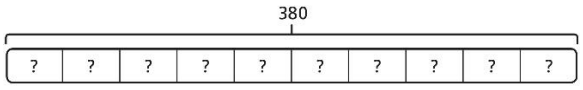
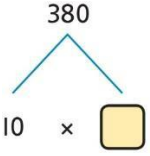
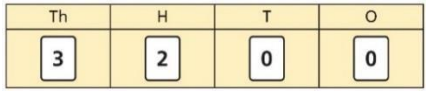
	Concrete	Pictorial	Abstract
	Year 5 Multiplication		
Squared and cubed numbers.	*Use cubes or counters to explore the earning of square numbers.* *25 is a square number because it is made from 5 rows of 5.* Use cubes to explore cube numbers.  8 is a cube number.	Use images to explore examples and non – examples of square numbers.  *8 × 8 = 64* *8^2 = 64*  *12 is not a square number, because you cannot multiply a whole number by itself to make 12.*	Understand the pattern of square numbers in the multiplication tables. Use a multiplication grid to circle each square number. Can children spot a pattern?
Multiplying by 10, 100 and 1,000		Understand the effect of repeated multiplication by 10. 	 *17 × 10 = 170* *17 × 100 = 17 × 10 × 10 = 1,700* 17 × 1,000 = 17 × 10 × 10 × 10 = 17,000

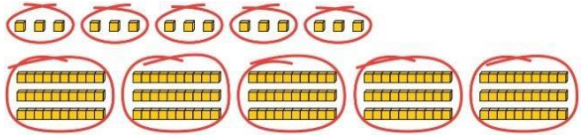
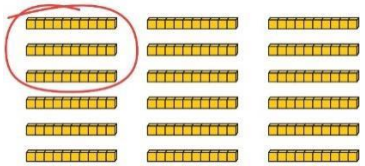
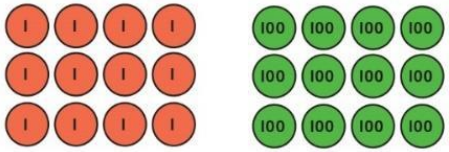
Multiplying by multiples of 10, 100 and 1,000	Use place value equipment to explore multiplying by unitising.  *5 groups of 3 ones is 15 ones.* *5 groups of 3 tens is 15 tens.* *So, I know that 5 groups of 3 thousands would be 15 thousands.*	Use place value equipment to represent how to multiply by multiples of 10, 100 and 1,000.  *4 × 3 = 12* *4 × 300 = 1,200*  *6 × 4 = 24* *6 × 400 = 2,400*	Use known facts and unitising to multiply. *5 × 4 = 20* *5 × 40 = 200* *5 × 400 = 2,000* *5 × 4,000 − 20,000* 5,000 × 4 = 20,000
Multiplying up to 4-digit numbers by a single digit	Explore how to use partitioning to multiply efficiently. *8 × 17 = ?*  8 × 10 = 80  8 × 7 = 56 80 + 56 = 136 *So, 8 × 17 = 136*	Represent multiplications, using place value equipment and add the 1's, then 10s, then 100s and then 1000s. 	 Use a column multiplication, including any required exchanges. 

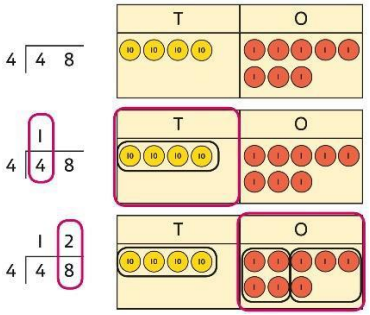
Multiplying 2-digit numbers by 2-digit numbers	Partition one number into 10s and 1s, then add the parts. *23 × 15 = ?*  10 × 15 = 150 10 × 15 = 150  3 × 15 = 45 There are 345 bottles of milk in total. H T O 1 5 0 1 5 0 + 4 5 3 4 5 1 *23 × 15 = 345*	*28 × 15 = ?*  H T O 2 0 0 1 0 0 8 0 + 4 0 4 2 0 1 *28 × 15 = 420*	3 4 × 2 7 2 3 8 34 × 7 6 8 0 34 × 20 3 4 × 2 7 2 3 8 34 × 7 6 8 0 34 × 20 9 1 8 34 × 27 1
Multiplying up to 4-digits by 2-digits		 143 × 12 = 1,716 There are 1,716 boxes of cereal in total. Th H T O 1 0 0 0 4 0 0 2 0 0 8 0 3 0 + 6 1 7 1 6 1 *143 × 12 = 1,716*	1 4 3 × 1 2 2 8 6 143 × 2 1 4 3 0 143 × 10 1 7 1 6 143 × 12 1 Progress to include examples that require multiple exchanges as understanding, confidence and fluency build. *1,274 × 32 = ?* *First multiply 1,274 by 2.*

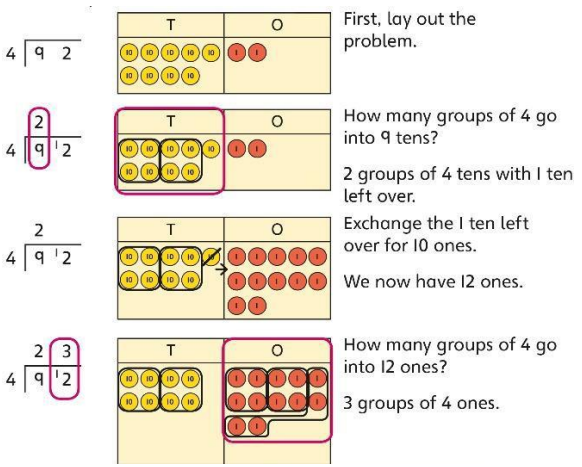
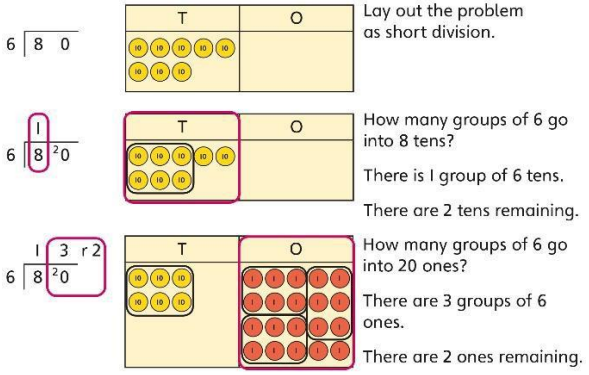
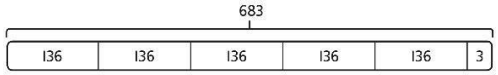
			1 2 7 4 × 3 2 2 5 4 8 1,274 × 2 *Then multiply 1,274 by 30.* 1 2 7 4 × 3 2 2 5 4 8 1,274 × 2 3 8 2 2 0 1,274 × 30 *Finally, find the total.* 1 2 7 4 × 3 2 2 5 4 8 1,274 × 2 3 8 2 2 0 1,274 × 30 4 0 7 6 8 1,274 × 32 *1,274 × 32 = 40,768*
Multiplying decimals by 10, 100 and 1,000	Use place value equipment to explore and understand the exchange of 10 tenths, 10 hundredths or 10 thousandths.	 0·14 × 10 = 1·4	2·5 × 10 = 25 2·5 × 100 = 250 2·5 × 1,000 = 2,500 Th H T O • Tth 2 • 5 2 5 • 2 5 0 • 2 5 0 0 •

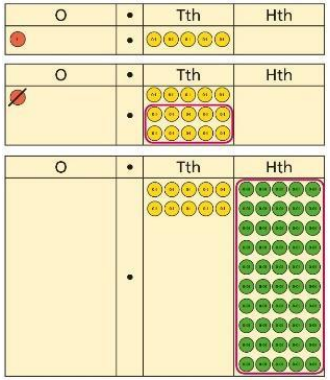
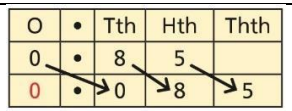
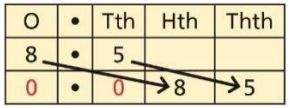
Multiply proper fractions and mixed numbers by whole numbers.	Use fraction circles to support their understanding. 	 	Children should be able to calculate the answer mentally or with support of jottings (as seen in pictorial stage) for difficult calculations.
Year 5 Division			
Understanding factors and prime numbers	 *24 ÷ 3 = 8* *24 ÷ 8 = 3* *8 and 3 are factors of 24 because they divide 24 exactly.*	Understand that prime numbers are numbers with exactly two factors. *13 ÷ 1 = 13* *13 ÷ 2 = 6 r 1* *13 ÷ 4 = 4 r 1*	*I know that 31 is a prime number because it can be divided by only 1 and itself without leaving a remainder.* *I know that 33 is not a prime number as it can be divided by 1, 3, 11 and 33.*

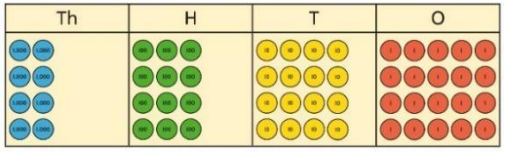
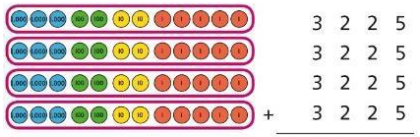
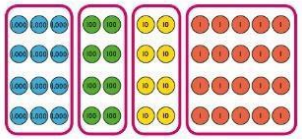
	24 ÷ 5 = 4 remainder 4.  5 is not a factor of 24 because there is a remainder.	*1 and 13 are the only factors of 13.* *13 is a prime number.*	*I know that 1 is not a prime number, as it has only 1 factor.*
Understanding inverse operations and the link with multiplication, grouping and sharing	*I have 28 counters.* *I made 7 groups of 4. There are 28 in total.* *I have 28 in total. I shared them equally into 7 groups. There are 4 in each group.* *I have 28 in total. I made groups of 4. There are 7 equal groups.*	Represent multiplicative relationships and explore the families of division facts.  *60 ÷ 4 = 15* *60 ÷ 15 = 4*	 12 ÷ 3 = 12 ÷ □ = 3 □ × 3 = 12 □ ÷ 3 = 12 12 □ × 3 Understand missing number problems for division calculations and know how to solve them using inverse operations. *22 ÷ ? = 2* *22 ÷ 2 = ?* *? ÷ 2 = 22* ? ÷ 22 = 2
Dividing whole numbers by 10, 100 and 1,000	*4,000 ÷ 1,000*  4,000 1,000 × □ *4,000 is 4 thousands.* *4 × 1,000= 4,000*	*380 ÷ 10 = 38*  380 ? ? ? ? ? ? ? ? ? ?  380 10 × □	 Th H T O 3 2 0 0 *3,200 ÷ 100 = ?* *3,200 is 3 thousands and 2 hundreds.* *200 ÷ 100 = 2* *3,000 ÷ 100 = 30*

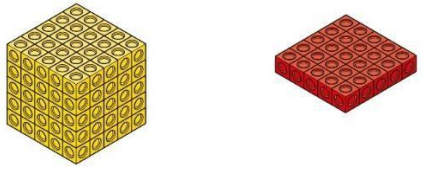
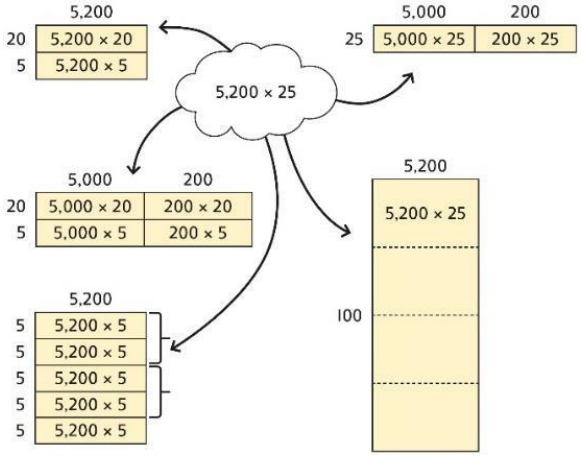
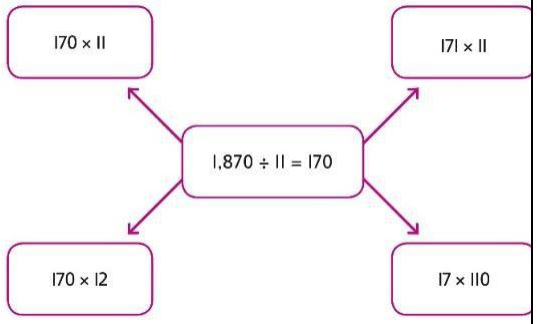
	So, 4,000 ÷ 1,000 = 4	*380 is 38 tens.* *38 × 10 = 380* *10 × 38 = 380* *So, 380 ÷ 10 = 38*	*3,200 ÷ 100 = 32* *So, the digits will move two places to the right.*
Dividing by multiples of 10, 100 and 1,000	Use place value equipment to represent known facts and unitising.  *15 ones put into groups of 3 ones. There are 5 groups.* *15 ÷ 3 = 5* *15 tens put into groups of 3 tens. There are 5 groups.* *150 ÷ 30 = 5*	Represent related facts with place value equipment when dividing by unitising.  *180 is 18 tens.* *18 tens divided into groups of 3 tens. There are 6 groups.* *180 ÷ 30 = 6*  *12 ones divided into groups of 4. There are 3 groups.* *12 hundreds divided into groups of 4 hundreds. There are 3 groups.*	Reason from known facts, based on understanding of unitising. Use knowledge of the inverse relationship to check. *3,000 ÷ 5 = 600* *3,000 ÷ 50 = 60* *3,000 ÷ 500 = 6* *5 × 600 = 3,000* *50 × 60 = 3,000* 500 × 6 = 3,000

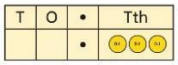
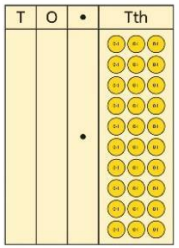
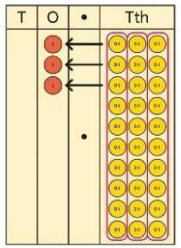
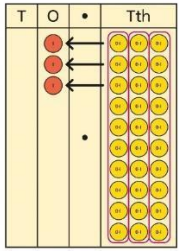
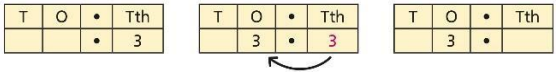
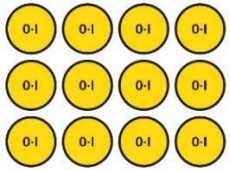
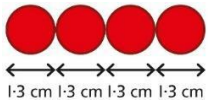
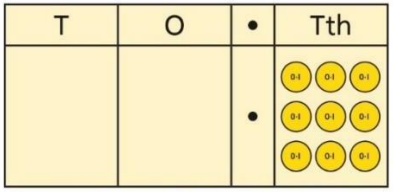
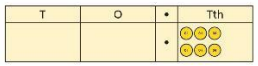
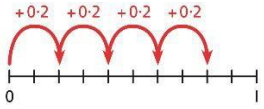
		1200 ÷ 400 = 3	
Dividing up to four digits by a single digit using short division	Explore grouping using place value equipment. *268 ÷ 2 = ?* *There is 1 group of 2 hundreds.* *There are 3 groups of 2 tens.* *There are 4 groups of 2 ones.* *264 ÷ 2 = 134*	Use place value equipment on a place value grid alongside short division. The model uses grouping. A sharing model can also be used, although the model would need adapting.  Lay out the problem as a short division. *There is 1 group of 4 in 4 tens.* *There are 2 groups of 4 in 8 ones.* Work with divisions that require exchange.	0 5 5 6 7 | 3 ^{3}8 ^{3}9 ^{4}2 *3,892 ÷ 7 = 556* Use multiplication to check. *556 × 7 = ?* *6 × 7 = 42* *50 × 7 = 350* *500 × 7 = 3500* *3,500 + 350 + 42 = 3,892*

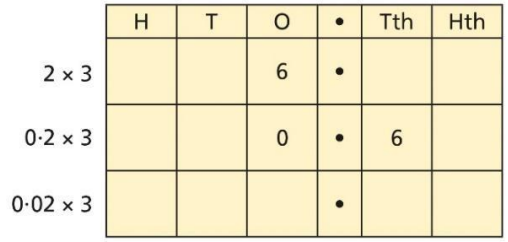
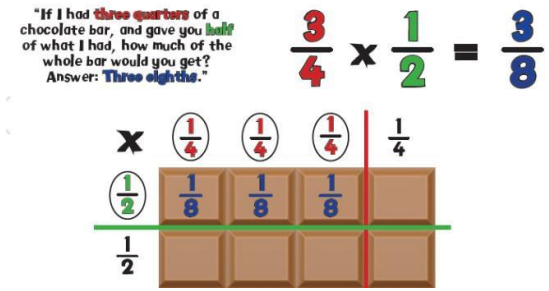
			
Understanding remainders	*80 cakes divided into trays of 6.*  *80 cakes in total. They make 13 groups of 6, with 2 remaining.*		In problem solving contexts, represent divisions including remainders with a bar model.  *683 = 136 × 5 + 3* *683 ÷ 5 = 136 r 3*
Dividing decimals by 10, 100 and 1,000	Understand division by 10 using exchange. *2 ones are 20 tenths.*	Represent division using exchange on a place value grid.	Understand the movement of digits on a place value grid.

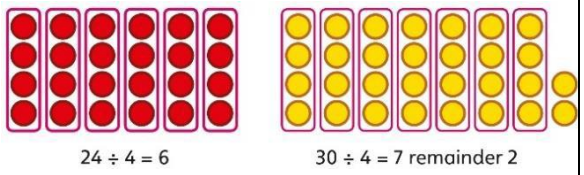
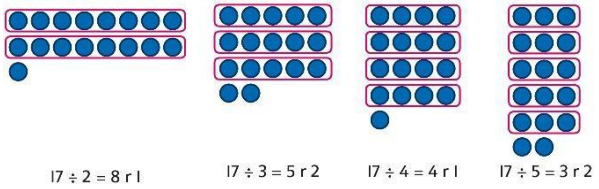
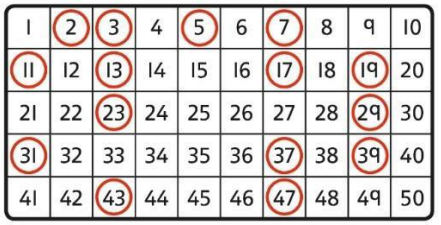
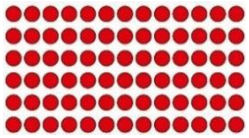
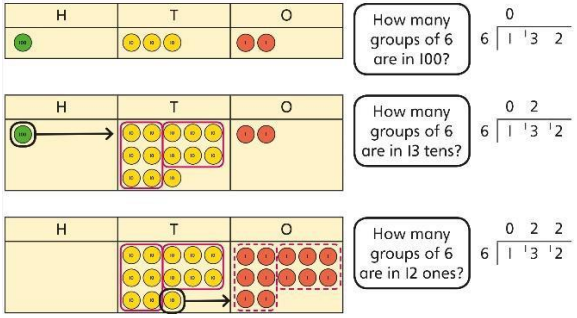
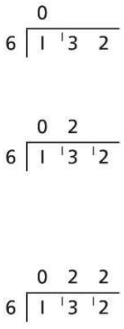
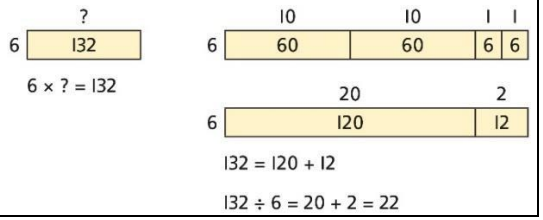
	20 tenths divided by 10 is 2 tenths.	 *1·5 is 1 one and 5 tenths.* *This is equivalent to 10 tenths and 50 hundredths.* *10 tenths divided by 10 is 1 tenth.* *50 hundredths divided by 10 is 5 hundredths.* *1·5 divided by 10 is 1 tenth and 5 hundredths.* 1·5 ÷ 10 = 0.15	 *0·85 ÷ 10 = 0·085*  8·5 ÷ 100 = 0·085

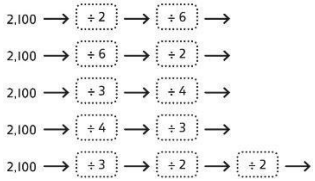
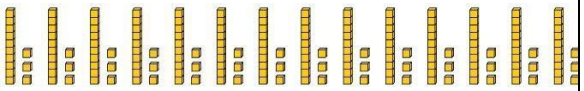
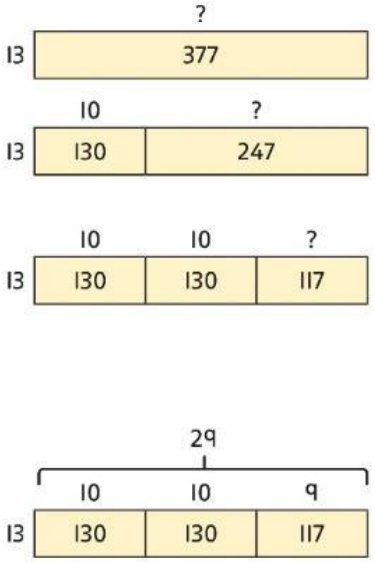
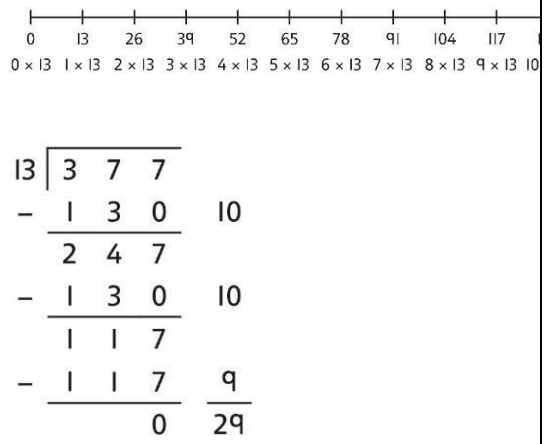
	Concrete	Pictorial	Abstract
Year 6 Multiplication			
Multiplying up to a 4-digit number by a single digit number	Use equipment to explore multiplications.  *4 groups of 2,345* *This is a multiplication:* *4 × 2,345* *2,345 × 4*	Use place value equipment to compare methods. **Method I**  3 2 2 5 3 2 2 5 3 2 2 5 + 3 2 2 5 1 2 9 0 0 1 1 2 **Method 2**  4 × 3,000 4 × 200 4 × 20 4 × 5 12,000 + 800 + 80 + 20 = 12,900	Compare and select appropriate methods for specific multiplications. **Method 3** 3,000 200 20 5 4 | 12,000 | 800 | 80 | 20 12,000 + 800 + 80 + 20 = 12,900 **Method 4** 3 2 2 5 × 4 1 2 9 0 0 1 2
Multiplying up to a 4-digit number by a 2-digit number		**Method I** 1,000 200 30 5 20 | 20,000 | 4,000 | 600 | 100 1 | 1,000 | 200 | 30 | 5	1 2 3 5 × 2 1 1 2 3 5 1 × 1,235 2 4 7 0 0 20 × 1,235 2 5 9 3 5 21 × 1,235

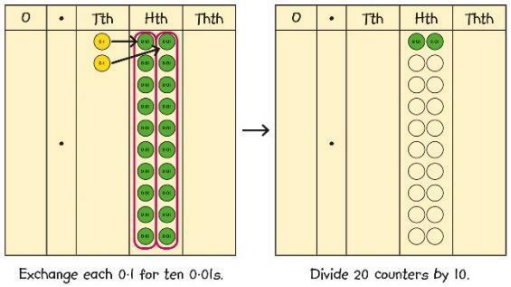
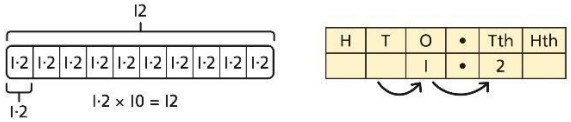
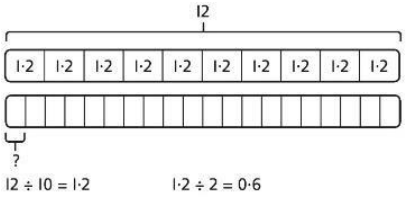
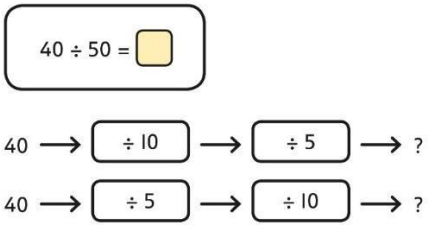
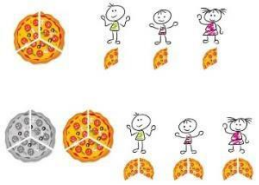
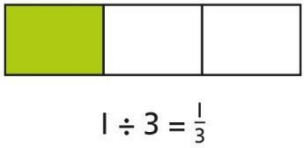
		1 2 3 5 × 2 1 5 — 1 × 5 3 0 — 1 × 30 2 0 0 — 1 × 200 1 0 0 0 — 1 × 1,000 1 0 0 — 20 × 5 6 0 0 — 20 × 30 4 0 0 0 — 20 × 200 2 0 0 0 0 — 20 × 1,000 2 5 9 3 5 — 21 × 1,235	
Using knowledge of factors and partitions to compare methods for multiplications	Use equipment to understand square numbers and cube numbers.  $5 \times 5 = 5^2 = 25$ $5 \times 5 \times 5 = 5^3 = 25 \times 5 = 125$	Compare methods visually using an area model. Understand that multiple approaches will produce the same answer if completed accurately. 	Use a known fact to generate families of related facts.  Use factors to calculate efficiently. 15×16 $= 3 \times 5 \times 2 \times 8$ $= 3 \times 8 \times 2 \times 5$ $= 24 \times 10$ $= 240$

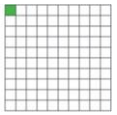
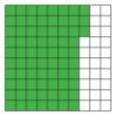
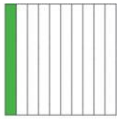
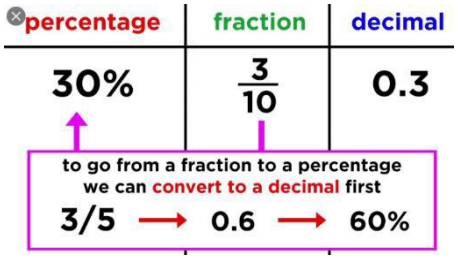
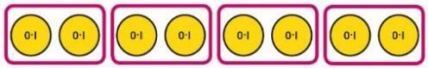
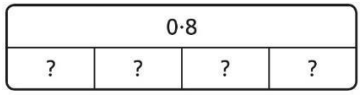
Multiplying by 10, 100 and 1,000	 Represent 0·3.  Multiply by 10.  Exchange each group of ten tenths. *0·3 × 10 = ?* *0·3 is 3 tenths.* *10 × 3 tenths are 30 tenths.* *30 tenths are equivalent to 3 ones.*	Understand how the exchange affects decimal numbers on a place value grid.   0·3 × 10 = 3	*8 × 100 = 800* *8 × 300 = 800 × 3* *= 2,400* *2·5 × 10 = 25* *2·5 × 20 = 2·5 × 10 × 2* *= 50*
Multiplying decimals	Explore decimal multiplications using place value equipment and in the context of measures.  *3 groups of 4 tenths is 12 tenths.* *4 groups of 3 tenths is 12 tenths.*  *4 × 1 cm = 4 cm*	Represent calculations on a place value grid. 3 × 3 = 9 3 × 0·3 = 0·9  Understand the link between multiplying decimals and repeated addition.  	Use known facts to multiply decimals. *4 × 3 = 12* *4 × 0·3 = 1·2* *4 × 0·03 = 0·12* *20 × 5 = 100* *20 × 0·5 = 10* *20 × 0·05 = 1* Find families of facts from a known multiplication. *I know that 18 × 4 = 72.* *This can help me work out:* *1·8 × 4 = ?*

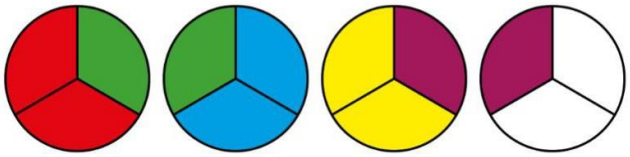
	4 × 0·3 cm = 1.2 cm *4 × 1·3 = 4 + 1·2 = 5·2 cm*		*18 × 0·4 = ?* *180 × 0·4 = ?* *18 × 0·04 = ?* Use a place value grid to understand the effects of multiplying decimals. 
Multiply simple pairs of proper fractions.		If I had 3/4 of a chocolate bar and gave you half, w much of the whole bar would you get?  	Children should be able to calculate the answer mentally or with support of jottings (as seen in pictorial stage).
Year 6 Division			
Understanding factors		Recognise prime numbers as numbers having exactly two factors. Understand the link with division and remainders.	Recognise and know primes up to 100.

	 *4 is a factor of 24 but is not a factor of 30.*		Understand that 2 is the only even prime, and that 1 is not a prime number. 
Dividing by a single digit	 *There are 78 in total.* *There are 6 groups of 13.* *There are 13 groups of 6.*		Use short division to divide by a single digit.  Use an area model to link multiplication and division. 

Dividing by a 2-digit number using factors	Understand that division by factors can be used when dividing by a number that is not prime.	Use factors and repeated division. *1,260 ÷ 14 = ?*  *1,260 ÷ 2 = 630* *630 ÷ 7 = 90* *1,260 ÷ 14 = 90*	Use factors and repeated division where appropriate. *2,100 ÷ 12 = ?* 
Dividing by a 2-digit number using long division	Use equipment to build numbers from groups.  *182 divided into groups of 13.* *There are 14 groups.*	Use an area model alongside written division to model the process. *377 ÷ 13 = ?*  *377 ÷ 13 = 29*	Use long division where factors are not useful (for example, when dividing by a 2-digit prime number). Write the required multiples to support the division process. *377 ÷ 13 = ?*  *377 ÷ 13 = 29*

Dividing by 10, 100 and 1,000	 *0·2 is 2 tenths.* *2 tenths is equivalent to 20 hundredths.* *20 hundredths divided by 10 is 2 hundredths.*	Understand the effect of dividing by 10, 100 and 1,000 on the digits on a place value grid.  Understand how to divide using division by 10, 100 and 1,000. *12 ÷ 20 = ?* 	Use knowledge of factors to divide by multiples of 10, 100 and 1,000.  *40 ÷ 5 = 8* *8 ÷ 10 = 0·8* *So, 40 ÷ 50 = 0·8*
Understanding the relationship between fractions and division	Use sharing to explore the link between fractions and division. *1 whole shared between 3 people.* *Each person receives one-third.* 	Use a bar model and other fraction representations to show the link between fractions and division. 	Use the link between division and fractions to calculate divisions. $5 \div 4 = \frac{5}{4} = 1\frac{1}{4}$ $11 \div 4 = \frac{11}{4} = 2\frac{3}{4}$

Calculate decimal equivalents		Using images children to convert between decimals, fractions and percentages. $\frac{1}{100} = 0.01 = 1\% =$  $\frac{73}{100} = 0.73 = 73\% =$  $\frac{1}{10} = 0.1 =$  $\frac{7}{10} = 0.7 =$ 	
Dividing decimals	 *8 tenths divided into 4 groups. 2 tenths in each group.*	Use a bar model to represent divisions.  $4 \times 2 = 8$ $\quad 8 \div 4 = 2$ So, $4 \times 0{\cdot}2 = 0{\cdot}8$ $\quad 0{\cdot}8 \div 4 = 0{\cdot}2$	$8\overline{)4 \cdot 24}$ $8\overline{)4 \cdot {}^{4}2\ 4}$ 0 · $8\overline{)4 \cdot {}^{4}2\ {}^{2}4}$ 0 · 5 $8\overline{)4 \cdot {}^{4}2\ {}^{2}4}$ 0 · 5 3

Divide proper fractions by whole numbers.	Children to use the fractions bars or fraction circles available in the resources room.		$3\frac{1}{3} \div \frac{2}{3} = 5$